Gravity-forward modelling of topography on the surface of the Moon by a combination of spatial- and spectral-domain techniques using 3D variable density

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Motivation

Gravitational information inferred from the shape and density of topography is useful, for instance, to determine the geoid or to approximate fine gravitational field features. When pushed to high resolutions, gravity-forward modelling becomes challenging, especially when evaluation points are located near the topography. This poster studies **accurate and efficient topographic gravitational field modelling near the topography while using 3D-variable density**.

The gravitational potential V of the topography H is given by Newton's integral

$$V(r,\Omega) = G \iint\limits_{\Omega'} \int\limits_{r'=R}^{R+H(\Omega')} \frac{\rho(r',\Omega')}{\ell(r,\psi,r')} (r')^2 \,\mathrm{d}r' \,\mathrm{d}\Omega'. \tag{1}$$

We assume the density ρ is square integrable in latitude and longitude and analytical in the radial direction, so that it can be expressed as

$$\rho(r',\Omega') = \sum_{i=0}^{\infty} \rho_i(\Omega')\,(r')^i, \qquad \rho_i(\Omega') = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \bar{\rho}_{nm}^{(i)}\,\bar{Y}_{nm}(\Omega'). \tag{2}$$

Eq. (1) can be evaluated in the spatial or spectral domain.

- **Spatial methods:** accurate near the topography, but slow.
- **Spectral methods:** efficient, but invalid near the topography if used globally.

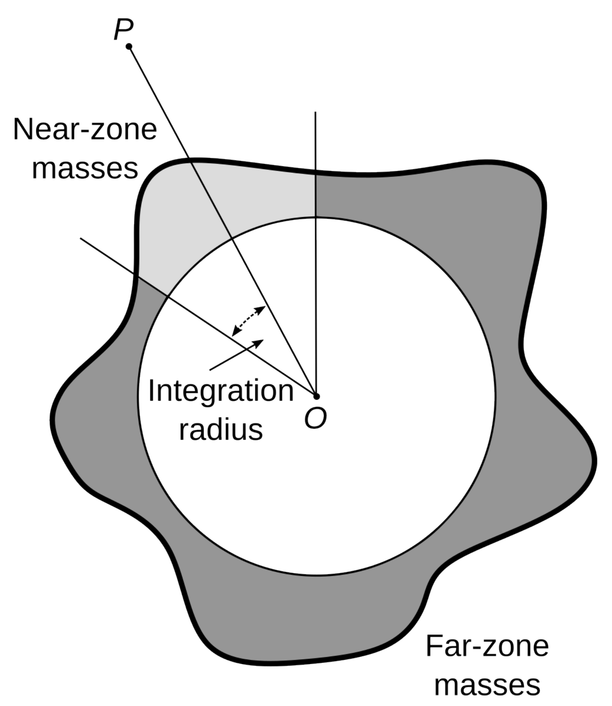
Sought is thus a combination of spatial and spectral techniques that retains the best from both worlds, the accuracy of spatial methods and the efficiency of spectral approaches.

We split the gravitational potential V into two constituents,

$$V = V^{\mathrm{Near}} + V^{\mathrm{Far}}, \tag{3}$$

where V^{Near} is the gravitational potential implied by masses up to some spherical distance ψ_0 from the evaluation point (near-zone masses) and V^{Far} is the contribution from the remaining far-zone masses.

We propose to use spatial-domain methods to get V^{Near} and spectral-domain methods modified to a spherical cap (hence not global) to get V^{Far}. If the hypothesis of [1] is true, the spectral method will converge even on the topography as long as the integration radius is larger than the largest topographic height $\max(H)$. This makes it possible to use the slow spatial-domain methods only within a small cap having a radius of a few km (in case of planetary bodies), while all the remaining far-zone masses are integrated using efficient FFT-based spectral methods.



This poster verifies the proposed combination of spatial- and spectral-domain methods in the framework of realistic 3D-variable densities.

Spatial-domain gravity-forward modelling

We evaluate the near-zone gravitational contributions using **tesseroids with density varying radially** as a finite-degree polynomial. Using different set of polynomial density coefficients for each tesseroid, 3D-variable densities can be gravity-forward modelled.

Applied is the method of [2], which relies on the Gauss–Legendre quadrature in the horizontal direction and analytical integration in the radial direction. In addition, 2D adaptive subdivision technique is applied for tesseroids close to evaluation points to ensure accurate results. [2] provide equations up the second-order directional derivatives of the gravitational potential.

Spectral-domain gravity-forward modelling

The far-zone gravitational potential is evaluated by the spectral method introduced by [3],

$$V^{\mathrm{Far}}(r,\Omega,\psi_0) = \frac{GM}{R}\sum_{n=0}^{N}\sum_{m=-n}^{n} \bar{V}_{nm}^{0,0,\mathrm{Far}}(r,\psi_0,R)\,\bar{Y}_{nm}(\Omega), \tag{4}$$

where

$$\bar{V}_{nm}^{u,v,\mathrm{Far}}(r,\psi_0,R) = (-1)^v\,\frac{2\pi\,R^{3+u}}{M}\sum_{p=1}^{P}\sum_{i=0}^{I} Q_{npi}^{u,v,\mathrm{Far}}(r,\psi_0,R)\,\overline{\mathrm{H}\rho}_{nm}^{(pi)}, \qquad u=0,1,2; \quad v=0,\dots,u, \tag{5}$$

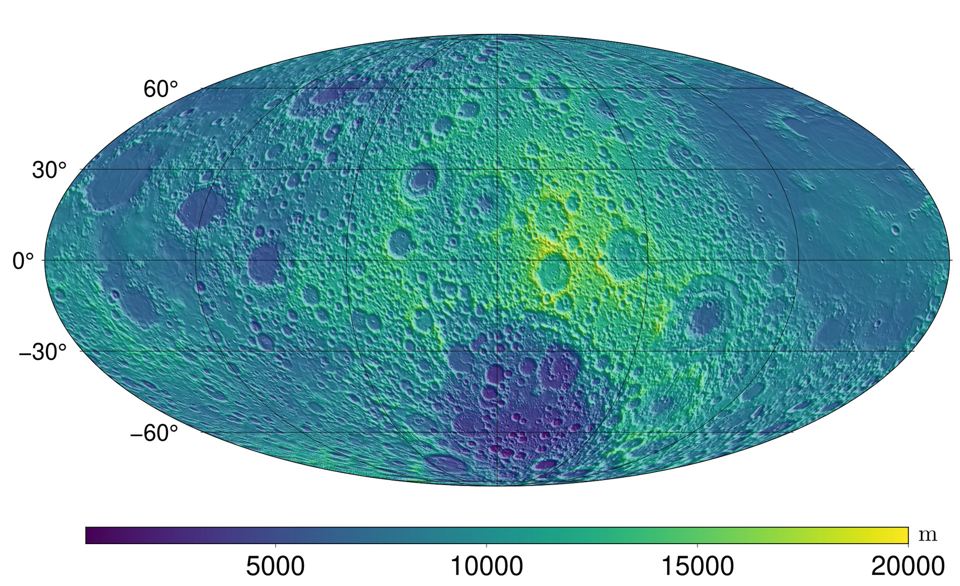
$$\overline{\mathrm{H}\rho}_{nm}^{(pi)} = \frac{1}{4\pi}\iint\limits_{\Omega'} \mathrm{H}\rho^{(pi)}(\Omega')\,\bar{Y}_{nm}(\Omega')\,\mathrm{d}\Omega', \qquad \mathrm{H}\rho^{(pi)}(\Omega') = \left(\frac{H(\Omega')}{R}\right)^p \rho_i(\Omega')\,R^i. \tag{6}$$

In Eq. (5), $Q_{npi}^{u,v,\mathrm{Far}}(r,\psi_0,R)$ are Molodensky's truncation coefficients. Equations for the first- and second-order directional derivatives of Eq. (4) are provided by [3].

Numerical implementation of Eq. (4) is available through CHarm, a C/Python library for high-degree spherical harmonic expansions (`https://www.charmlib.org`).

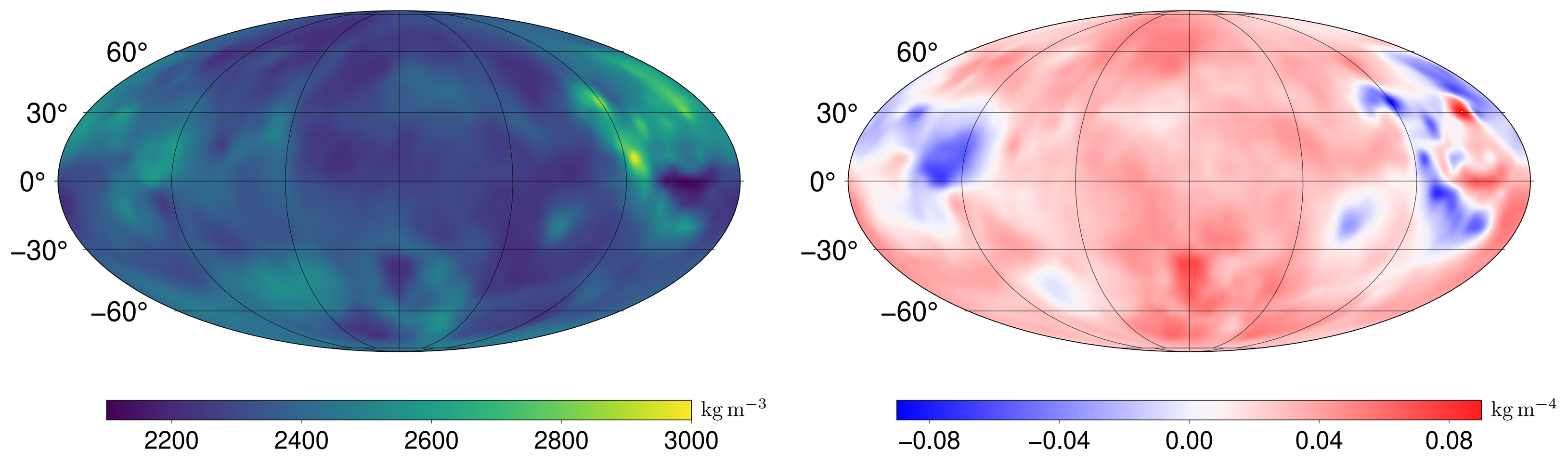
Data

- **Topography:** Moon_LDEM128_shape_pa_11519 [4] up to degree $N_H = 2160$ referenced to $R = 1{,}728{,}000$ m



Data (continuation)

- **Density:** LIN_L250-650_TC40 [5] up to degrees $N_{\rho_0} = N_{\rho_1} = 2160$ (originally $1^\circ \times 1^\circ$ grids, but the transformation to Eq. (2) gives rise to additional harmonics). Original grids are shown here



Setup

- Gravitational quantities: V, V^{x}, V^{y}, V^{z}, V^{xx}, V^{xy}, V^{xz}, V^{yy}, V^{yz}, V^{zz}
- Evaluation points: 5 arcmin grid at 0.1 m height above the topography
- Integration radius: $\psi_0 = 10^\circ$ (threshold for convergence: $\sim 0.66^\circ$)
- Polynomial density order: $I = 1$
- Spatial gravity-forward modelling: $\Delta\varphi = \Delta\lambda = 30$ arcsec, 2nd order Gauss–Legendre quadrature
- Spectral gravity-forward modelling: $P = 15$, $N = 10{,}800$, 50 radial derivatives for the downward continuation from a Brillouin sphere

Results

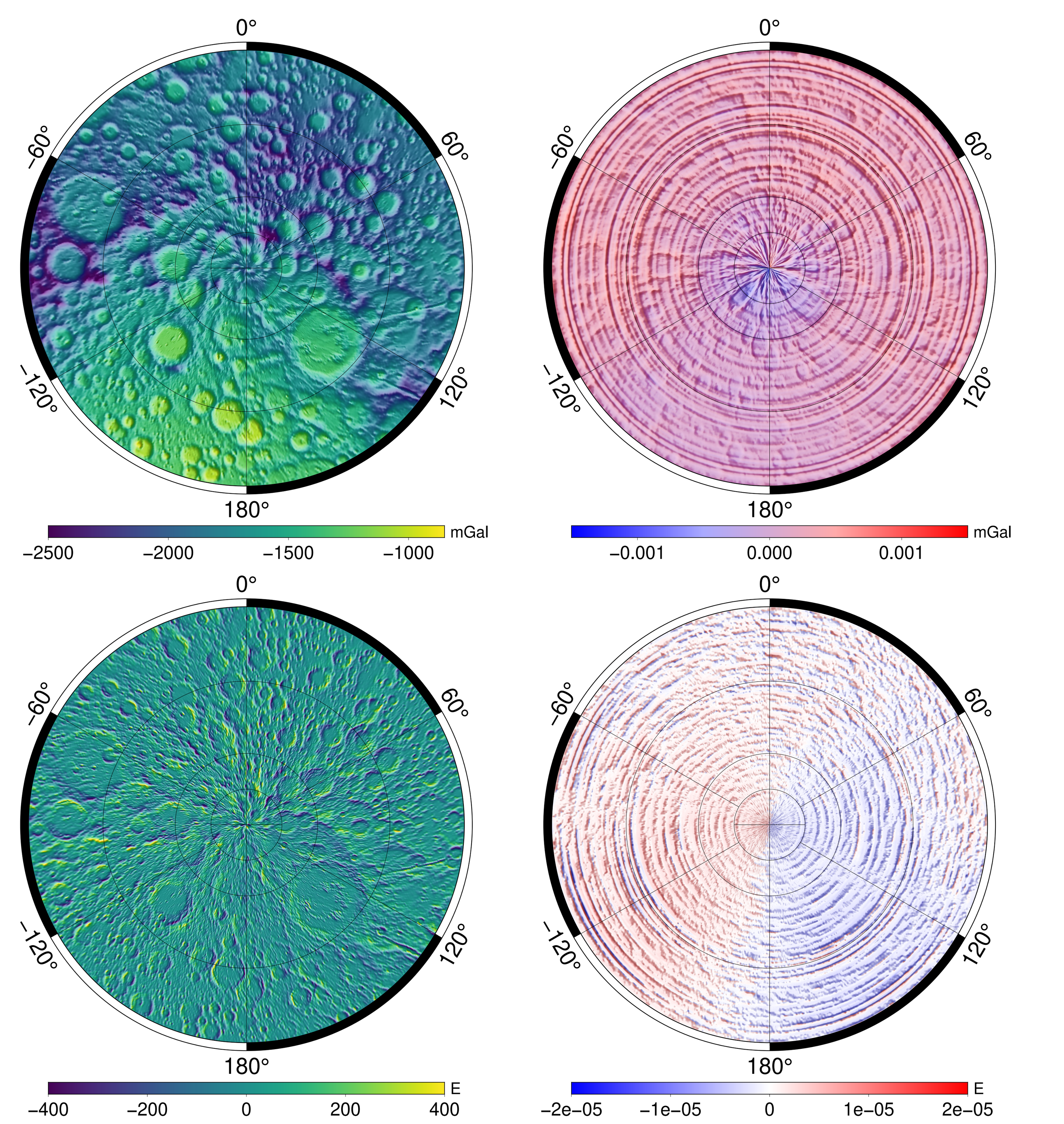


Figure 1. Validation over the south pole up to latitude -60°. *Left column:* Reference values; *Right column:* Errors of the new method; *Top row:* V^{z}; *Bottom row:* V^{yz}

Summary

- Gravitational field of topographies with 3D-variable density can be computed efficiently on the topography by combining spatial- and spectral-domain methods
- Challenge for future research is to prove that the integration radius for far-zone effects needs to be larger than the largest topographical height to ensure convergence of the spectral method modified to spherical caps (details in [1])

References

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Acknowledgements

Part of the research results was obtained using the computational resources procured in the national project National competence centre for high performance computing (project code: 311070AKF2) funded by European Regional Development Fund, EU Structural Funds Informatization of society, Operational Program Integrated Infrastructure. We thank the HPC centre at the Slovak University of Technology in Bratislava, which is a part of the Slovak Infrastructure of High Performance Computing (SIVVP project, ITMS code 26230120002, funded by the European region development funds, ERDF). The poster template is due to `https://github.com/anishathalye/gemini`.

Funding

Funded by the EU NextGenerationEU through the Recovery and Resilience Plan for Slovakia under the project No. 09I03-03-V04-00273.